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# Effect of Uniform Suction on MHD Flow Through Porous Medium Due To A Rotating Disk

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Abstract— The present paper is devoted to the study of the magnetohydrodynamic (MHD) flow of an incompressible viscous and electrically conducting fluid due to an infinite porous rotating disk at small distance from a porous medium. A uniform suction is applied through the surface of the disk. The domain of flow is divided into two regions: the free fluid region between the disk and the porous medium and the porous region. The governing equations of motion, in terms of cylindrical polar coordinates, are reduced to a set of nonlinear ordinary differential equations by similarity transformations and then solved by using the approximation method. The solutions are obtained by solving Navier-Stokes equations in the free fluid region, and Brinkman equations in the porous region with adequate boundary conditions at the interface. Graphical representation of the results are outlined for different values of Hartmann number, suction parameter and the porosity of the medium. The effect of these parameters upon the velocity fields are examined. The torque acted on the disk have been also computed. The main result of the present work is that, the presence of the magnetic field effects on the velocity field in both flow regions. This effect depends on the suction process. It is also noticed that the magnetic field reduces the velocity components, while suction process increases them. Therefore, the torque due to viscous friction acting on the disk increases with increasing the magnetic field strength.

**Index Terms**—MHD, Porous medium, rotating disk, suction, Reynolds number , Hartmann number, Brinkman equations.

## 1 INTRODUCTION

Since many decades, flow of a viscous fluid through and past a porous medium have been the subject of intensive studies, specially at recent years, due to its many engineering and scientific applications. Examples of these applications are extraction process of fluid from the porous ground and in lubrication of porous bearings [1]. Moreover, It has many applications in biomedical and chemical engineering for filtration and purifications processes. Cunningham and Williams [2] had reported several geophysical applications of flow in porous medium, viz. porous roller and its natural occurrence in the flow of rivers through porous banks and the flow of oil through underground porous rocks.

Probably for the first time, the flow due to an infinite plane rotating disk was discussed by Karman [3]. He suggested a tractable method to transform the set of partial differential equations governing the flow to nonlinear ordinary differential equations to be simple in mathematical handling. The flow due to a rotating infinite radius disk with uniform suction at the disk has been discussed by Stuart [4] and obtained numerical solutions for small values of suction and asymptotic solutions for large values of suction. Rizvi [5] examined the MHD flow over a rotating disk in the presence of weak magnetic field. Effects of an axial magnetic field on the flow about a rotating disk were studied also by Kakutani [6]. Pande [7] analyzed a series solution for the effects of an axial magnetic field and suction (injection) on the flow about an insulated rotating disk in the presence of strong suction and a weak magnetic fields. Purohit and Bansal [8] considered the flow of an incompressible viscous and incompressible electrically conducting fluid between two rotating and a stationary naturally permeable disks. Ariel [9] discussed the numerical behavior of MHD flow in the vicinity of a rotating disk. Attia [10] considered time varying rotating disk flow and heat transfer of a conducting fluid with suction or injection.

Darcy [11] initiated the theory of flowing through a porous medium. For the steady flow, he assumed that viscous force were in equilibrium with external forces due to pressure difference and body forces. Later on Brinkman [12] proposed modification to Darcy's law of porous medium. In most of these examples, the flow field is divided into two regions, namely (I) free fluid region, and (II) porous region, where the fluid flows through a porous medium. To link flows in the two regions, certain matching conditions are required at the interface of the two regions. This type of couple flows, with different geometries and with several kinds of matching conditions, has been examined by several authors, viz. William [13] and Ochoa-Tapia et al. [14], [15]. Steady flow between a rotating and a stationary naturally permeable disks had been studied by Verma and Bhatt [16]. Srivastava et al. [17] studied the flow in a porous medium induced by torsional oscillation of a disk near its surface. The flow of viscous incompressible fluid confined between a rotating disk and a porous medium was analyzed by Chaudhary et al. [18].

An analysis has been made to investigate the effects of uniform magnetic field on the forced flow of a conducting viscous fluid through a porous medium induced by a rotating disk; Sharma et al. [19]. Recently, Dufour and Soret effects on unsteady MHD convective heat and mass transfer flow due to a rotating disk, has been investigated by Maleque [20].

From this survey, it is clear that the problem of fluid flow generated within a porous medium by a rotating disk near it is being more significant for application in technology. Hence, in the present analysis, it is proposed to study the effects of uniform suction on MHD flow through a porous medium due to a rotating disk. The region between them is filled with an in-

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compressible electrically conducting viscous fluid. We confined our attention for the effects of an axial magnetic field and suction on both the velocities and the torque on the disk in the two domain flow regions.

## 2 Formulation of the Problem

We consider the steady flow of an incompressible viscous electrically conducting fluid confined between a rotating disk and a porous medium. The cylindrical polar coordinates $(r, \theta, z)$ are the most suitable system of coordinates for the present problem. So, let the disk of radius r lie in the plane $z = d$ and rotates uniformly with constant angular velocity $\Omega$ about the z-axis perpendicular to its own plane; figure 1. Mass transfer from the fluid may take place at the disk surface by direct suction. The rate of mass removal $w_o$ is uniform at all points on the disk surface. An external uniform magnetic field with constant intensity $B_o$ is applied perpendicular to the surface of the disk, i.e. $\underline{B} = B_o \hat{z}$ . The induced magnetic field is assumed to be small in comparison with the applied magnetic field.

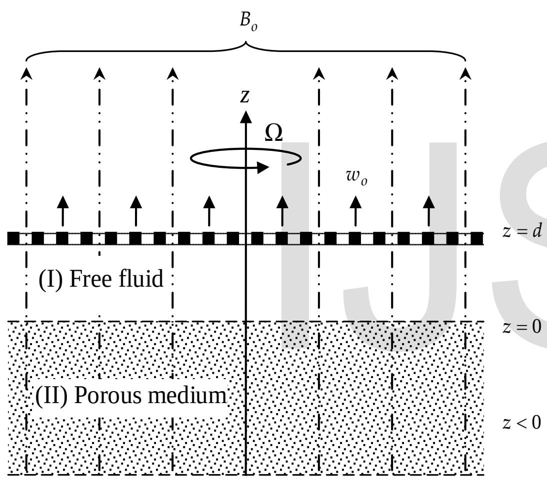


Fig. 1. Physical model and coordinates.

The steady flow field in the free region is governed by the Navier-Stokes equation; namely,

$$\rho(\underline{\upsilon} \cdot \nabla)\underline{\upsilon} = -\nabla p + \mu \nabla^2 \underline{\upsilon} + \underline{J} \times \underline{B} \,, \qquad (1)$$

where $\rho$ is the density, $\underline{\upsilon}$ the velocity vector, $p$ pressure, $\mu$ the coefficient of viscosity, $\nabla^2$ the Laplacian operator, $\underline{J}$ the current density vector and $\underline{B}$ the magnetic field vector. The porous medium is fully saturated with fluid. The magnitude of velocity components in the porous medium are very small so their squares and higher powers are neglected and a term accounting for the resistance by porous material is added. Hence, we get the following Brinkman equation [12] which governs the flow of a viscous fluid in a porous medium:

$$-\nabla P + \mu_e \nabla^2 \underline{V} - \frac{\mu}{k}\underline{V} + \underline{J} \times \underline{B} = 0 \,, \qquad (2)$$

where $\underline{V}$ is the velocity vector, $P$ the pressure, $\mu_e$ the effective viscosity of Brinkman flow model, and $k$ is the permeability of porous medium.

## 3 Basic Governing Equations

As mentioned above, we divide the flow region into two zones. Zone I is the region defined as $(0 \le z \le d)$ in which the fluid flows freely and its motion is governed by Navier-Stokes equation, (1). Zone II $(z \le 0)$ is the region in which the fluid flowing through the pores of porous material and its motion is governed by Brinkman equation, (2). The interface between the two zones is at $z = 0$ . The velocity components $(u, \upsilon, w)$ in the free region and $(U, V, W)$ in the porous region are taken to be in the direction of $(r, \theta, z)$ respectively.

The Navier-Stokes and continuity equations for the MHD flow in region I, are given by:

$$\rho\left[\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)u - \frac{\upsilon^2}{r}\right] = -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 u - \frac{u}{r^2}\right) - \sigma B_o^2 u \,, \qquad (3a)$$

$$\rho\left[\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\upsilon + \frac{u\upsilon}{r}\right] = \mu\left(\nabla^2 \upsilon - \frac{\upsilon}{r^2}\right) - \sigma B_o^2 \upsilon \,, \qquad (3b)$$

$$\rho\left[u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right]w = -\frac{\partial p}{\partial z} + \mu \nabla^2 w \,. \qquad (3c)$$

The Brinkman and continuity equations in region II are given by:

$$-\frac{\partial P}{\partial r} + \mu_e\left(\nabla^2 U - \frac{U}{r^2}\right) - \frac{\mu}{k}U - \sigma B_o^2 U = 0 \,, \qquad (4a)$$

$$\mu_e\left(\nabla^2 V - \frac{V}{r^2}\right) - \frac{\mu}{k}V - \sigma B_o^2 V = 0 \,, \qquad (4b)$$

$$-\frac{\partial P}{\partial z} + \mu_e \nabla^2 W - \frac{\mu}{k}W = 0 \,, \qquad (4c)$$

$$\frac{\partial}{\partial r}(rU) + \frac{\partial}{\partial z}(rW) = 0 \,. \qquad (4d)$$

The appropriate boundary conditions of the aforementioned systems are:

$$\left.\begin{array}{l} u = 0 \\ \upsilon = r\Omega \\ w = w_o \end{array}\right\} \; at \quad z = d, \qquad \left.\begin{array}{l} U \to 0 \\ \\ V \to 0 \end{array}\right\} \; as \quad z \to -\infty \,. \qquad (5a)$$

Conditions at the interface of the porous medium and free fluid, $z = 0$ , have been investigated by Ochoa-Tapia and Whitaker [14], [15], show that the equations require a discontinuity in the shearing stresses while retaining the continuity of the velocity components and the normal stress. Hence, the following conditions at the interface are required:

$$\left.\begin{array}{l} u = U, \;\; \upsilon = V, \;\; w = W, \;\; p = P \\ \mu_e \dfrac{\partial U}{\partial z} - \mu \dfrac{\partial u}{\partial z} = \beta \dfrac{\mu}{\sqrt{k}} U \\ \mu_e \dfrac{\partial V}{\partial z} - \mu \dfrac{\partial \upsilon}{\partial z} = \beta \dfrac{\mu}{\sqrt{k}} V \end{array}\right\} \; at \quad z = 0 \,, \qquad (5b)$$

where $\beta$ is a dimensionless constant of order one with positive or negative sign.

The set of partial differential equations encountered in (3) through (5) represents a formidable mathematical problem. We seek the solutions of (3) and (4) under the boundary conditions (5), in the following form:

In zone I:

$$u = r\Omega\, f'(\eta), \; \upsilon = r\Omega\, g(\eta), \; w = -2d\Omega\, f(\eta), \; p = -\mu\,\Omega\, p_1(\eta) \qquad (6a)$$

In zone II:

$$U = r\Omega F'(\eta),\ V = r\Omega G(\eta),\ W = -2d\Omega F(\eta),\ P = -\mu\Omega P_1(\eta)\ , \quad (6b)$$

On substituting (6a) and (6b) into (3) to (5), we obtain the following set of equations:

In zone I:

$$f''' - m^2 f' = \mathrm{Re}\left(f'^2 - 2ff'' - g^2\right), \quad (7a)$$

$$g'' - m^2 g = 2\mathrm{Re}\left(f'g - fg'\right), \quad (7b)$$

$$p_1' = 4\mathrm{Re}\, ff' + 2f''\ . \quad (7c)$$

In zone II:

$$F''' = \alpha^2 F'\ , \quad (8a)$$

$$G'' = \alpha^2 G\ , \quad (8b)$$

$$P_1' = 2\gamma F'' - 2\delta^2 F\ , \quad (8c)$$

where $\eta = z/d$ is a non-dimensional distance along the axis of rotation and the prime denotes differentiation with respect to $\eta$. The corresponding boundary conditions become:

$$\left.\begin{array}{l} f' = 0 \\ g = 1 \\ f = S \end{array}\right\} \ at \quad \eta = 1, \qquad \left.\begin{array}{l} F' \to 0 \\ \\ G \to 0 \end{array}\right\} \ as \quad \eta \to -\infty\ . \quad (9)$$

The physical parameters appearing in (7) to (9) are:

$$\left.\begin{array}{lll} S = -\dfrac{2d\Omega}{w_o}, & \delta = \dfrac{d}{\sqrt{k}}, & \mathrm{Re} = \dfrac{\rho\Omega d^2}{\mu}, \\ \gamma = \sqrt{\dfrac{\mu_e}{\mu}}, & m = \sqrt{\dfrac{\sigma B_o^2 d^2}{\mu}}, & \alpha = \sqrt{\dfrac{\delta^2 + m^2}{\gamma^2}}. \end{array}\right\} \quad (10)$$

where $S$ is the suction parameter, $\delta$ the Darcy number, Re the Reynolds number, $\gamma$ the dimensionless viscosity and $m$ the magnetic induction parameter or Hartmann number. Using the expressions of velocity components in both zones, the matching conditions, (5b), at the interface can be respectively written as:

$$\left.\begin{array}{l} f = F, \quad f' = F', \quad g = G \\ f'' = \gamma^2 F'' - \beta\delta F' \\ g' = \gamma^2 G' - \beta\delta G \end{array}\right\} \ at \quad \eta = 0\ . \quad (11)$$

Notice that, (7c) and (8c) are decoupled from (7a) and (8a), and once $f(\eta)$ and $F(\eta)$ have been determined, $p_1(\eta)$ and $P_1(\eta)$ can be obtained by solving (7c) and (8c).

## 4 SOLUTION OF THE PROBLEM

The solution of the problem requires the determination of the velocity field components $\underline{v}$ and $\underline{V}$ from (7) and (8) subject to the boundary conditions (9) and (11); respectively.

In zone II, the solution of (8a) and (8b) satisfying the boundary conditions 9 are given by:

$$\left.\begin{array}{l} F(\eta) = \dfrac{A}{\alpha} e^{\alpha\eta} + C, \\ G(\eta) = B e^{\alpha\eta}. \end{array}\right\} \quad (12)$$

The constants $A$, $B$ and $C$ are to be determined by matching conditions at the interface, $\eta = 0$.

In zone I, the solution of the governing equations are obtained by the perturbation method. The Reynolds number Re, defined in terms of the angular velocity $\Omega$, is assumed to be small; i.e. $\mathrm{Re} << 1$. Therefore, the unknown functions, $f$ and $g$, can be expanded in successive powers of Re as the following:

$$\left[f,\ g\right] = \sum_{n=0}^{\infty} \mathrm{Re}^n \left[f^{(n)},\ g^{(n)}\right], \quad (13)$$

In matching $f$ and $g$ with $F$ and $G$ at the interface, the constants $A$, $B$ and $C$ can be expanded in powers of Re as:

$$\left[A,\ B,\ C\right] = \sum_{n=0}^{\infty} \mathrm{Re}^n \left[A_n,\ B_n,\ C_n\right], \quad (14)$$

Accordingly, the solution of (7a), (7b), (8a) and (8b) are given as:

In zone I:

$$f(\eta) = S + \mathrm{Re}\left[\frac{k_3}{m}\left(e^{m\eta} - e^{m}\right) - \frac{k_4}{m}\left(e^{-m\eta} - e^{-m}\right) + \frac{d_9}{2m}\left(e^{2m\eta} - e^{2m}\right) + d_{10}(\eta - 1) - \frac{d_{11}}{2m}\left(e^{-2m\eta} - e^{-2m}\right)\right], \quad (15a)$$

$$g(\eta) = a_1 e^{m\eta} + a_2 e^{-m\eta} + \mathrm{Re}\left[\left(k_1 + d_3\eta\right)e^{m\eta} + \left(k_2 + d_4\eta\right)e^{-m\eta}\right] \quad (15b)$$

In zone II:

$$F(\eta) = S + \mathrm{Re}\left(\frac{A_1}{\alpha} e^{\alpha\eta} + C_1\right), \quad (16a)$$

$$G(\eta) = \left(B_o + \mathrm{Re}\, B_1\right) e^{\alpha\eta}\ , \quad (16b)$$

where the values of the constants $A$, $B$ and $C$ satisfying the boundary conditions 9 and matching conditions 11 at the interface have been calculated and they are given below.

$$a_1 = \frac{1}{\Delta_1}(m + a), \qquad a_2 = \frac{1}{\Delta_1}(m - a),$$

$$d_3 = -\frac{S}{\Delta_1}(m + a), \qquad d_4 = -\frac{S}{\Delta_1}(m - a),$$

$$d_9 = -\frac{1}{3}\left(\frac{m + a}{m\Delta_1}\right)^2, \qquad d_{10} = \frac{m^2 - a^2}{m^2 \Delta_1^2},$$

$$d_{11} = -\frac{1}{3}\left(\frac{m - a}{m\Delta_1}\right)^2, \qquad d_{12} = -\left(d_9 e^{2m} + d_{10} + d_{11} e^{-2m}\right),$$

$$d_{13} = -(2m - a)d_9 + a d_{10} + (2m + a)d_{11}\ ,$$

$$k_1 = \frac{S}{\Delta_1^2}\left[(m + a)^2 e^{m} + \left(m^2 - a^2 + 2m\right) e^{-m}\right],$$

$$k_2 = \frac{S}{\Delta_1^2}\left[\left(m^2 - a^2 - 2m\right)e^{m} + (m - a)^2 e^{-m}\right],$$

$$k_3 = \frac{1}{\Delta_1}\left[(m + a)d_{12} + d_{13}e^{-m}\right], \qquad k_4 = \frac{1}{\Delta_1}\left[(m - a)d_{12} - d_{13}e^{m}\right],$$

$$A_1 = k_3 + k_4 + d_9 + d_{10} + d_{11}, \qquad B_o = \frac{2m}{\Delta_1},$$

$$B_1 = k_1 + k_2\ ,$$

$$C_1 = \frac{k_3}{m}\left(1 - e^{m}\right) - \frac{k_4}{m}\left(1 - e^{-m}\right) + \frac{d_9}{2m}\left(1 - e^{2m}\right) - d_{10} - \frac{d_{11}}{2m}\left(1 - e^{-2m}\right) - \frac{A_1}{\alpha},$$

$$a = \gamma^2 \alpha - \beta\delta\ ,$$

$$\Delta_1 = (m + a)e^{m} + (m - a)e^{-m}\ .$$

## 5 RESULTS AND DISCUSSION

The system of non-linear ordinary differential equatins (7), (8) is solved under the conditions given by (9) and (11) for the

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three components of the flow velocity and pressure. From the solution of the present problem it is clear that, the MHD flow depends on four parameters; namely, the magnetic parameter $m$, the suction parameter $S$, Darcy number δ and the dimensionless viscosity $\gamma$. For $S=0$ we have the case of an impermeable rotating disk. Insight into the physical occurrences within the flow field can be obtained by studying of the velocity profiles.

### 5.1 Velocity Distributions

Firstly, we turn our attention to the distribution of the axial velocities $F(\eta)$ and $f(\eta)$ in porous and free zones respectively. The effect of the magnetic parameter $m$ on the axial velocity are shown in figures 2a and 2b. In these figures, $\mathrm{Re}=0.2$, $\delta=3$, $\gamma=1.5$ and $\beta=0.5$. The negative values of the axial velocity indicate an inflow from the porous medium toward the interface and then toward the disk surface. Consider now the case of an impermeable disk; $S=0$. The rotating disk acts like as a fan, drawing fluid axially inward from the surroundings toward the disk surface. However, because the surface is solid, the inward fluid finds its path blocked, so it must be turned and discharges in the radial direction where there is no obstruction. Thus there is a close correlation between the axial inflow and the radial outflow. So, in figure 2a, we see that the negative velocity of inflow starting from its largest value at large negative $\eta$ and decreasing steadily as we approach the disk due to fluid escape into the radial direction.



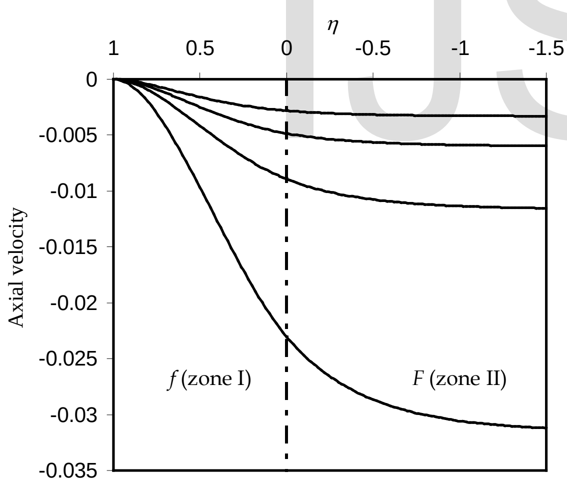


Fig. 2a. Axial velocities f and F at $S=0$, $m=0.5, 1, 1.5, 2$ (bottom to top).

Consider now the application of the suction process at the disk surface, $S=-2$ as shown in figure 2b. Beside the fanlike pumping of the rotating disk, there is an additional pumping due to the suction. So, the quantity of fluid drawn in from the surroundings increases. The inflowing fluid has two parts; it may continue through the disk's holes which is being rather possible, specially at high suction, or it may reroute into the radial direction. As a consequence, $F(\eta)$ tends to become almost constant as we enter the porous medium, $\eta\to-\infty$. It is clear that, figures 2a and 2b, the effect of the magnetic parameter $m$ on the axial velocity is to increase the velocities in both zones. Hence it is concluded that magnetic field enhances the fluid motion.

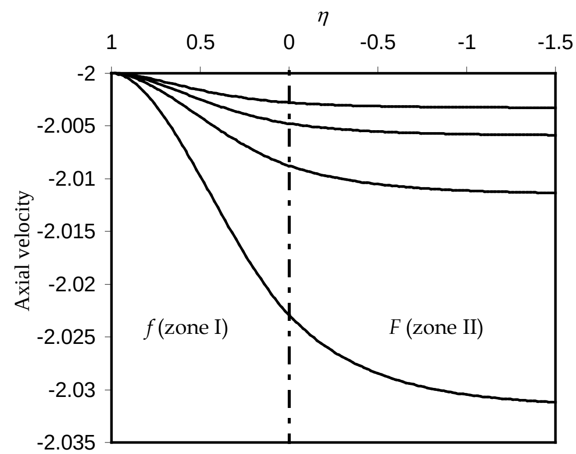


Fig. 2b. Axial velocities f and F at $S=-2$, $m=0.5, 1, 1.5, 2$ (bottom to top).

Figure 3 shows the behavior of the radial velocities $f'$ and $F'$ against $\eta$ in both zones I and II respectively for various values of $m$ and $S=-2$. Since the radial velocity is zero at the disk surface and in ambient fluid, there must be a maximum value denoted by $f'_{\max.}$ in zone I. This maxima is being positive because of the radial flow is always outward along the disk.

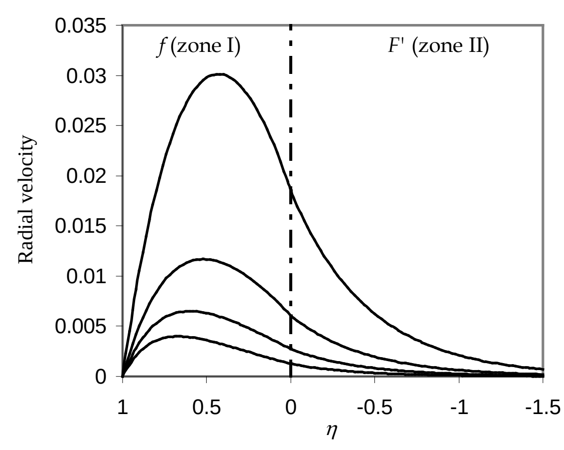


Fig. 3. Radial velocities $f'$ and $F'$ at $S=-2$, $m=0.5, 1, 1.5, 2$ (top to bottom).

Owing to figure 3, we see that the radial velocity increases monotonically with deceasing $\eta$ until reaching maxima, and then decays exponentially as we enter the porous medium until vanishing at a large distance from the interface, $\eta\to-\infty$.

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As the value of $m$ decreases the maxima $f'_{\max.}$ moves away from the disk and the magnitude of the radial velocities $f'$ and $F'$ increase.

The tangential velocity is directly driven through the action of viscosity by the rotation of the disk. Figure 4a shows the distribution of the tangential velocities $g$ and $G$ as a function of $\eta$ for different values of $m$ at $S = 2$. The magnetic field induces a magnetic force in the tangential direction which opposes the tangential fluid velocity. So, the tangential velocities are everywhere reduce with increasing $m$. From figure 4b we see that, the suction process gives rise to the profile points.

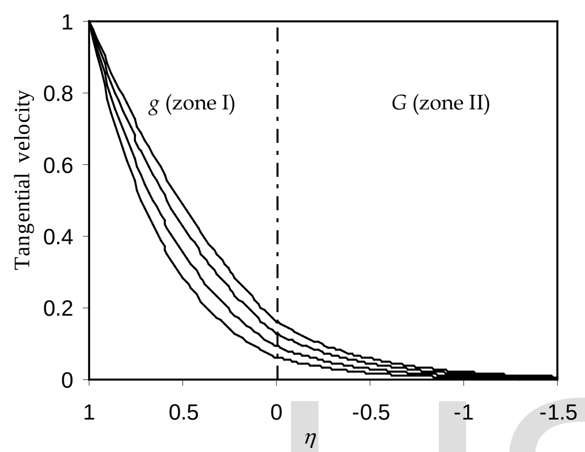


Fig. 4a. Tangential velocities g and G at $S = 2$, $m = 0.5, 1, 1.5, 2$ (top to bottom).

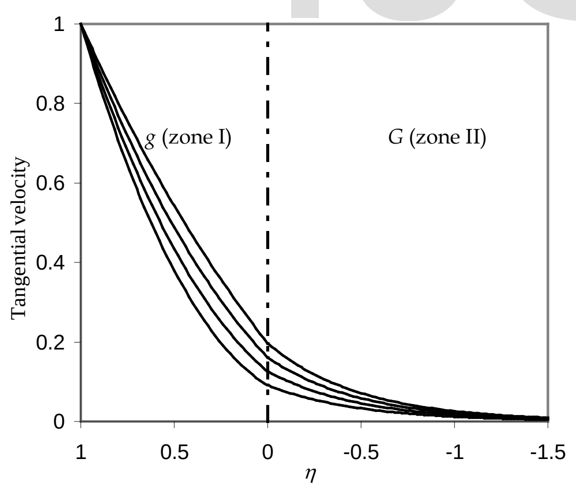


Fig. 4b. Tangential velocities g and G at $m = 0.5$, $S = -1, -2, -3, -4$ (top to bottom).

### 5.2 Shear Stress And Torque

From the practical point of view, a torque is required to maintain a steady rotation of the disk which is the action of viscosity in the fluid adjacent to the disk. Such a torque is needed to overcome the tangential shear stress imposed by the fluid on the disk surface. The tangential shear $\tau_{z\theta}$ at the surface, $z = d$, is achieved by applying the Newtonian shear formula:

$$\tau_{z\theta} = \mu\left(\frac{\partial \upsilon}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right)\bigg|_{z=d}. \tag{17a}$$

In terms of the transformation variables of (6a) and (6b), this becomes

$$\tau_{z\theta} = \frac{\mu \Omega r}{d} g'(1), \tag{17b}$$

where $g'(1)$, is found from the solution given in (15b) as:

$$g'(1) = m\left(a_1 e^{m} - a_2 e^{-m}\right) + \mathrm{Re}\left\{\left[m(k_1 + d_3) + d_3\right]e^{m} - \left[m(k_2 + d_4) - d_4\right]e^{-m}\right\}. \tag{17c}$$

The torque $M$ required to overcome the tangential shear on one side of the rotating disk [21] is

$$M = \int_0^{r_o} 2\pi r^2 \left(\tau_{z\theta}\right)_{z=d} dr, \tag{18a}$$

where $r_o$ is the disk radius. Introducing (17b) and integrating, we get

$$M = \frac{\pi \mu \Omega r_o^4}{2d} g'(1). \tag{18b}$$

Therefore, both tangential shear $\tau_{z\theta}$ and the torque $M$ are proportional to the slope $g'(1)$ of the tangential velocity profile; Table 1. It is shown that the effect of suction is to increase the tangential shear and the torque requirements. These results reflect the change in tangential velocity profile as previously discussed.

Moreover, there is also a surface shear stress, $\tau_{rz}$, in the radial direction. Again, using the Newton shear relations and then introducing the variables of the analysis, we have:

$$\tau_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\bigg|_{z=d}, \tag{19a}$$

or,

$$\tau_{rz} = \frac{\mu \Omega r}{d} f''(1). \tag{19b}$$

Another quantity of interest is the inflow $F(-\infty)$ induced in the fluid of porous medium; i.e. region II by the motion of the disk. These values, which constitute the horizontal portions of the profiles of figures 2a and 2b, have been listed in Table 1. The strong MHD effect which gives rise to a sharply decreased flow is clear evidently in the table. The table shows that $g'(1)$ increase with an increasing $S$, $m$, $\gamma$ and $\delta$ while $F(-\infty)$ increases with increasing $S$ but decrease with increasing $m$, $\gamma$ and $\delta$. It is observed that, if we take $\gamma = 1$, $\beta = 0$, $m = 0$ and $S = 0$ in our analysis and $\phi = \lambda = 1$, $\alpha = 0$ in the work of Srivastava and Barman [22], the results of both the studies are comparable. Further, it is noted that if we take $S = 0$ and $m = 0$ in our analysis, the results reduced to that of Chaudhary et al. [18].

TABLE 1 EFFECT OF $S$, $M$, $\gamma$, AND $\delta$ ON $g'(1)$ AND ON AXIAL VELOCITY AT INFINITY $F(-\infty)$

## 6 CONCLUSION

In the present work, the steady MHD flow of a viscous incom-

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pressible electrically conducting fluid due to an infinite porous rotating disk at small distance from a porous medium is investtigated. A uniform suction is applied through the surface of the disk. It is clear that the flow depends on the parameters

| $S$ | $M$ | $\gamma$ | $g'(1)/\delta$ | | $-F(-\infty)/\delta$ | |
|---|---|---|---|---|---|---|
| | | | 3 | 5 | 3 | 5 |
| 0 | 0.5 | 1.2 | 0.7971 | 0.8823 | 0.0314 | 0.0267 |
| | | 2 | 0.9194 | 0.9753 | 0.0318 | 0.0259 |
| | 1 | 1.2 | 1.1123 | 1.1662 | 0.0121 | 0.0098 |
| | | 2 | 1.1951 | 1.2327 | 0.0109 | 0.0089 |
| -1 | 0.5 | 1.2 | 0.9795 | 1.0737 | 1.0314 | 1.0267 |
| | | 2 | 1.1137 | 1.1728 | 1.0318 | 1.0259 |
| | 1 | 1.2 | 1.3012 | 1.3603 | 1.0121 | 1.0098 |
| | | 2 | 1.3912 | 1.4309 | 1.0109 | 1.0089 |
| -2 | 0.5 | 1.2 | 1.1618 | 1.2650 | 2.0314 | 2.0267 |
| | | 2 | 1.3080 | 1.3704 | 2.0318 | 2.0259 |
| | 1 | 1.2 | 1.4901 | 1.5543 | 2.0121 | 2.0098 |
| | | 2 | 1.5874 | 1.6291 | 2.0109 | 2.0089 |

$S$, $m$, $\delta$ and $\gamma$. The table and figures presented above show the effect of these parameters on the fluid flow. By looking into the details of the flow field, the following trends are strongly evident in all of these figures:

1. The rotation of a disk near a porous medium extracts the fluid from the porous medium.
2. As the magnetic field strength increases, the fluid velocity decreases.
3. For any value of the magnetic parameter $m$, axial velocity toward the disk increases with increasing of the suction parameter.
4. The axial velocity component at a large distance from the interface does not vanish. Therefore, a boundary layer is formed at the interface.
5. Torque due to viscous fraction acting on the disk increases with increasing the magnetic field strength for any value of suction.

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